External Validation of the Strategy Choice Model for Addition

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The study was designed to assess the external validity, both convergent and discriminant forms, of the strategy choice model proposed by Siegler and Shrager (1984, in C. Sophian, Ed., Origins of cognitive skills (pp. 229–293), Hillsdale, NJ: Erlbaum). Forty-two preschool/kindergarten children were administered the Wechsler Preschool and Primary Scale of Intelligence (WPPSI) and the Arithmetic subtest of the Wide Range Achievement Test (WRAT) and were videotaped as they solved 25 simple addition problems. Strategies, and their associated reaction times, used in problem solving were classified in accordance with the strategy choice model. A variable which represented adaptive strategy choices for solving addition problems was significantly correlated with the Arithmetic subtest of the WRAT and with the Arithmetic, Geometric Design, and Mazes subtests of the WPPSI. Component scores for a memory retrieval variable were significantly related to performance on both tests of arithmetic ability; the greater the facility of information retrieval from long-term memory the better the performance on the traditional measures. This pattern of correlations suggested strategy choices for solving addition problems were related to both the numerical and spatial ability domains, whereas the speed of executing the component process of fact retrieval was related only to arithmetic ability. In all, the study provided strong support for the convergent validity of the strategy choice model and modest support for its discriminant validity. Finally, the relationship between information-processing models and reference ability measures was discussed.

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Young children may employ one of a variety of alternative strategies to solve any single addition equation. These potential strategies range from the use of one of many counting strategies (Baroody, 1987; Carpenter & Moser, 1984; Groen & Parkman, 1972; Groen & Resnick, 1977) to the retrieval of an answer from a long-term memory network of addition facts (Ashcraft, 1982, 1987; Ashcraft & Fierman, 1982; Geary, Widaman, Little, & Cormier, 1987; Kaye, Post, Hall, & Dineen, 1986; Siegler, 1987; Siegler & Robinson, 1982; Siegler & Shrager, 1984). The distribution of associations model which has been proposed by Siegler and Shrager (1984) provides a useful conceptual framework for the representation of how children select different strategies to solve different small number addition problems.

The primary purpose of the present study was to attempt to replicate the basic findings of Siegler and his colleagues (Siegler & Robinson, 1982; Siegler & Shrager, 1984) and to concurrently assess the external validity (Sternberg, 1977) of the strategy choice model for addition. Finally, by examining the relationship between individual differences in strategy choices for solving addition problems, the speed of executing specific processes, and performance on psychometric measures of arithmetic ability, the study attempted to isolate the source of individual differences on these traditional ability tests.

Within Siegler’s (1986) model, four basic strategies have been identified (Siegler & Robinson, 1982). Of these four strategies, three are visible or audible, overt strategies, and are labeled (a) counting fingers—the children use their fingers to physically represent the addends of the problem and then count their fingers to reach a sum; (b) fingers—the children use their fingers to represent the addends but do not visibly count them before giving an answer; and (c) counting—the children count audibly or move their lips as if counting implicitly. The fourth strategy was termed no visible strategy. Here, the children provide an answer without the use of their fingers or by visibly counting. This final strategy is thought to reflect a memory retrieval process (Geary, 1987; Siegler & Shrager, 1984).

Siegler and Shrager (1984) formally integrated these strategies within a single conceptual framework, with the proposal of the distribution of associations model of strategy choice. Here, strategy choice is primarily governed by the distribution of associations between an addition problem and all potential answers to that problem. More precisely, the strategy selected for problem solving is a function of the associative strength between the problem and its correct answer, which is indexed by the probability of correctly retrieving that answer, and a confidence criterion. “The confidence criterion defines a value that must be exceeded by the associative strength of a retrieved answer for the child to state that answer” (Siegler & Shrager, 1984, p. 239).
When solving an addition problem, the child first sets two parameters: (a) the confidence criterion and (b) a search length time which indicates the maximum number of retrieval attempts a child will make before that child invokes an alternative strategy. Retrieval is then attempted. Retrieval efforts continue as long as the value of the confidence criterion exceeds the associative strength of each retrieved answer, and as long as the number of searches does not exceed the value of the search length parameter (Siegler & Shrager, 1984). If the retrieval process fails to yield a satisfactory answer, the child may then make a sophisticated guess by stating the final retrieved answer (Siegler, 1986). Otherwise, the child resorts to an overt or internal elaboration strategy. For example, in an overt elaboration the child puts up her fingers to represent the problem's addends. This strategy augments the associative strength between the problem and its correct answer. Following the elaboration, if the answer's associative strength now exceeds the confidence criterion, the child answers. If not, the child finally resorts to some algorithmic process, such as counting fingers, to reach a solution.

The strength of the strategy choice model is that with a single underlying mechanism, the probability of correct retrieval combined with the confidence criterion, it can represent many of the strategy choices children make when solving addition problems. To illustrate, Siegler (1988a) recently identified three groups of children who differed on several basic characteristics of strategy choices in solving addition, subtraction, and word identification problems. These three groups of children were termed "good" students, "not-so-good" students, and "perfectionists." The groups of "good" students and "perfectionists" did not differ in mean mathematics ability as assessed by traditional achievement measures but both of these groups showed a significantly higher mean standing than did the group of "not-so-good" students. However, the "good" students and the "perfectionists" differed in terms of the distribution of strategies they employed to solve the presented problems.

Basically, for the solution of the arithmetic problems, the "perfectionists" employed the retrieval strategy less frequently than they used a counting strategy. but when they did state a retrieved answer it was almost always the correct answer. The "good" students relied on the retrieval strategy for the solution of the majority of the presented arithmetic problems but tended to make more retrieval errors than did the "perfectionists." Finally, the "not-so-good" students also employed retrieval as a primary strategy for solving arithmetic problems; however, these students made significantly more retrieval errors than did the "good" students.

Within the strategy choice model, the observed strategy differences of the children comprising each of these three groups can be understood in terms of individual differences in the probability of retrieving a correct answer and the stringency of the associated confidence criterion. Siegler
argued that both the "perfectionists" and "good" students likely had rather peaked distributions of associations, but differed in the rigor of their confidence criterion. The "perfectionists" relative to the "good" students appeared to require a higher level of confidence in the retrieved answer before that answer was stated. The "not-so-good" students, relative to the other two groups, appeared to have both less peaked distributions of associations and a rather lenient confidence criterion. Thus, these students tended to state retrieved answers even when those answers were likely to be incorrect. In all, these group differences in observed strategy choices were easily represented by Siegler's (1986) model, with the inferred differences in the peakedness of the distributions of associations and by differences in the rigor of the confidence criterion.

Moreover, these group differences suggest that there is not a direct one-to-one relationship between overt behavior, and by inference cognitive processes, and ability as assessed by traditional psychometric measures. Thus, the frequent use of memory retrieval as a strategy for solving arithmetic problems does not, in itself, indicate high mathematics ability or an advance in the development of cognitive skills. Rather, the results of Siegler's (1988a) study suggest that, at least for children, ability as measured by traditional measures might be more strongly related to the adaptive use of alternative strategies than to the frequent use of the developmentally "most mature" strategy. The strategy choice model provides a useful framework for the understanding of adaptive (or maladaptive) strategy choices and may, therefore, provide a useful model for relating individual differences in strategy choices to performance on traditional ability measures.

Nevertheless, the ultimate utility of the model for the representation of strategy choices across many cognitive domains and for identifying the source of individual differences on traditional psychometric ability measures will depend upon the demonstration of the internal and external validity of the model (Sternberg, 1977). Internal validation may take either of two forms, intensive and extensive validity. Intensive validity requires the demonstration that experimental findings, e.g., observed strategies, are logically and mathematically consistent with the predictions of the conceptual model. Extensive validity is established by the demonstration that highly similar, or identical, strategies are invoked for the solution of theoretically similar tasks (Keating & Hobbitt, 1978). Experiments conducted by Siegler and his colleagues have provided support for the internal validity, both intensive and extensive forms, of the strategy choice model (Siegler, 1986, 1988a,b; Siegler & Robinson, 1982; Siegler & Shrager, 1984; Siegler & Taraban, 1986).

External validation (Sternberg, 1977) of the model would require the demonstration of high correlation between a strategy choice parameter, or parameters, and reference ability measures of the same cognitive skill
(Geary & Widaman, 1987; Sternberg & Gardner, 1983). More rigorously, external validation studies need to demonstrate both convergent and discriminant validity (Campbell & Fiske, 1959) of the model. Convergent validity would be demonstrated with high correlation between a variable representing strategy choices and traditional measures of the same cognitive skill, whereas discriminant validity would be demonstrated with low correlation between this same strategy choice parameter and traditional measures of different cognitive skills.

In the present study, the strategies and associated reaction times (RT) employed by 42 preschool/kindergarten children to solve simple addition problems were recorded and classified as one of the four strategies identified by Siegler and Robinson (1982), and these same children were administered the Arithmetic subtest of the Wide Range Achievement Test (WRAT: Jastak & Jastak, 1965) and the Wechsler Preschool and Primary Scale of Intelligence (WPPSI: Wechsler, 1967) which includes an Arithmetic subtest and nine additional subtests. The Arithmetic subtest of the WRAT, for children of this age, includes up to 13 items. The first six of these items require counting, number identification, and an understanding of cardinal value. The final seven items comprise simple addition and subtraction problems with numbers less than 10, and four of these seven problems (e.g., 5 − 3) are solved with the use of pencil and paper. The items comprising the arithmetic subtest of the WPPSI assess a broader range of mathematics skills and includes up to 20 items. The first eight of these items require counting or a simple understanding of quantity (i.e., more or less), whereas the remaining 12 items require the mental solution of word problems which require addition or subtraction. To illustrate, consider the following problem, which is of intermediate difficulty. “Bob ate 1 piece of candy, Sue ate 2 pieces, and Jack ate 2 pieces. All together, how many pieces of candy did they eat?” (Wechsler, 1967, p. 62).

The convergent validity of the strategy choice model would be demonstrated if scores on a variable representing strategy choices in addition were significantly correlated with performance on both of the just mentioned arithmetic ability measures. The discriminant validity of the strategy choice model would be demonstrated if scores on the strategy choice variable were not related to performance on any of the remaining nine WPPSI subtests, or if the magnitude of the correlation between the strategy choice variable and the two arithmetic tests was significantly higher than the level of correlation between this variable and the remaining nine WPPSI subtests.

Finally, the pattern of correlations between a variable, or variables, representing strategy choices in addition, the temporal duration of strategy execution, and performance on the arithmetic tests should provide information as to the source of individual differences on these traditional
psychometric measures. To illustrate, it might be the case that individual differences in strategy choices, the speed of executing specific processes, or some combination of the two underlie ability differences as measured by traditional tests (Hunt, 1983). The present study will empirically address this issue for arithmetic.

METHOD

Subjects

Forty-two (26 male, 16 female) 4- (n = 10), 5- (n = 22), or 6- (n = 10) year-old preschool or kindergarten children served as subjects. The subjects had a mean age of 66.4 months (SD = 7.7) and a mean Full Scale IQ of 116.5 (SD = 11.9).

Stimuli and Apparatus

All subjects were individually administered the Wechsler Preschool and Primary Scale of Intelligence (Wechsler, 1967) and the Arithmetic subtest of the Wide Range Achievement Test (Jastak & Jastak, 1965) and were videotaped as they solved 25 addition problems.

Addition stimuli consisted of the 25 problems consisting of all possible pairs of the integers 1 to 5. Five random orders of problem presentation were generated, and eight or nine subjects were presented the problems using each of the random orders.

Equipment included a Panasonic WV-1150A TV camera and a Panasonic NV-8050 Time Lapse video recorder. The experimenter controlled the timer, which recorded reaction time with an accuracy of ±0.1 s. This level of accuracy seemed to be adequate given the relatively long solution times for children of this age (cf. Siegler & Robinson, 1982). The timer was started with the presentation of the addend and stopped when the child verbally produced an answer.

Procedure

Each subject was tested individually and videotaped as each problem was solved. Subjects were tested in a partitioned section of their classroom, or in an adjacent room, and were initially presented with instructions identical to those used by Siegler and Robinson (1982).

I want you to imagine that you have a pile of oranges. I’ll give you more oranges to add to your pile; then you need to tell me how many oranges you have altogether. Okay? You have m oranges, and I’m going to give you n to add to your pile. How many do you have altogether? (p. 290).

Following the presentation of the first several problems each child was asked if they would prefer the problems to be presented in the form of “how much is m + n?”; nearly all of the children responded “yes.”
The majority of problems were, therefore, presented in this form. Subjects were given a rest period after the presentation of the 13th problem or if they appeared to be fatigued or distracted.

During the test period, the strategy used to solve each problem was recorded by the experimenter, and each was classified as one of the four strategies described by Siegler and Robinson (1982): (a) counting fingers, (b) fingers, (c) verbal counting, or (d) no visible strategy. Tapes for 13 subjects were later reviewed by an independent observer and strategy choice was recorded for each problem. The agreement between observers was 92.0%.

RESULTS AND DISCUSSION

For ease of presentation, the results and the discussion is presented in three sections. The first section describes aggregate level analyses on strategy characteristics—replication. The second section presents correlations between scores on a variable representing strategy choices in addition and performance on the ability measures, whereas the final section examines the relationship between component scores for a memory retrieval variable and performance on the ability tests.

Replication

The basic characteristics of strategy selection are presented in Table 1. The second column of Table 1 presents the percentage of trials on which each of the strategies was used. With the exception of the fingers strategy, the frequency of strategy use was highly comparable to the findings of Siegler and Robinson (1982), and based on these findings it was hypothesized that the mean solution times comparing independent strategies would differ significantly. These solution times are presented in the third column of Table 1, and, as found by Siegler and Robinson, mean solution times were the lowest for the retrieval strategy followed, in turn, by the fingers, verbal counting, and counting fingers strategies, respectively.

The mean solution time for the retrieval strategy was significantly lower than the mean solution time for the fingers strategy, \( t(22) = 5.49, p < .001 \), and the mean solution time for the fingers strategy was, in turn, significantly lower than the mean solution time for the verbal counting strategy, \( t(20) = 3.37, p < .01 \). However, the mean solution times for the verbal counting and counting fingers strategies did not differ significantly, \( t(22) = 0.34, p > .05 \). Finally, overall levels of accuracy and the characteristics of addition errors were also consistent with the findings of Siegler and Robinson, as noted in the final two columns of Table 1.

Correlations among variables representing the frequency of strategy use, collated across the 25 experimental problems, and indices of problem difficulty are presented in Table 2. Of particular interest was the relationship
between the probability that any given problem was answered correctly using the memory retrieval strategy and the frequency with which one of the overt strategies was used to solve the same problem. As predicted by the strategy choice model, variables representing the above probabilities were significantly and inversely related, \( r = .73, p < .0001 \). As the probability of correct retrieval increased, the probability of visible strategy use decreased.

Moreover, the probability of correct retrieval for these 3 problems was inversely related to the ranked difficulty (rank: Wheeler, 1939) of the same problems, \( r = .74, p < .0001 \), and showed a strong positive correlation with the associative strength (AS) values provided by Siegler (1986), \( r = .88, p < .0001 \). The remaining correlations presented in Table 2 are also consistent with predictions of the strategy choice model, although the strength of the relationships are somewhat lower than those reported by Siegler and Robinson (1982).

**Strategy Choices and Performance on the Ability Measures**

The frequency with which each of the 42 subjects in the study used each of the four strategies was collated, and standard scores for the Arithmetic subtest of the WRAT and standard scores for each of the 10 WPPSI subtests, as well as the three IQ scores (Verbal IQ, Performance IQ, and Full Scale IQ) were obtained.

Examination of individual protocols indicated that 25 subjects never used the fingers strategy and only 4 of the remaining 17 subjects used the strategy on more than three trials; therefore, information for the
fingers strategy was not used in any subsequent analyses. Furthermore, 40 of the 42 subjects used the retrieval strategy at least once but only two of these 40 subjects correctly retrieved addition facts for all 35 problems. Further examination of individual protocols indicated that 37 subjects correctly retrieved addition facts on at least one trial. For the remaining trials, subjects also tended to invoke the retrieval strategy but were making errors, or they employed one of the two counting (verbal counting or counting fingers) strategies.

The foregoing observations were consistent with the finding that the number of incorrect retrieval trials was negatively correlated with the proportion of verbal counting trials, \( r = -0.31, p < .05 \), and with the proportion of counting fingers trials, \( r = -0.34, p < .05 \). Finally, the proportion of verbal counting trials was unrelated to the proportion of counting fingers trials, \( r = 0.00, p > .50 \). In other words, 93% of the subjects who used the retrieval strategy were able to retrieve a correct answer from long-term memory at least once, but for the solution of the remaining problems some of these subjects tended to guess, i.e., retrieve incorrect answers, whereas the remaining subjects tended to use either the verbal counting strategy or the counting fingers strategy.

Based on these results, and Siegler’s (1988a) recent findings, a theoretically and empirically justifiable (Keating & MacLean, 1987) strategy choice variable, which was a composite of two variables, was constructed. The first of these two variables was coded as follows: (number of correct no visible strategy trials/number of total no visible strategy trials). A high score on this variable would indicate that when the retrieval strategy was employed it produced a correct answer. The second variable, which

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### TABLE 2

<table>
<thead>
<tr>
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<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1. Visible trials</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>2. Correct retrieval</td>
<td>—0.73</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>3. Retrieval errors</td>
<td>0.58</td>
<td>—0.98</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>4. AS</td>
<td>—0.58</td>
<td>0.88</td>
<td>0.89</td>
<td>—</td>
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<td>—</td>
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<tr>
<td>5. Sum</td>
<td>0.25</td>
<td>—0.66</td>
<td>0.69</td>
<td>—0.81</td>
<td>—</td>
<td>—</td>
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<tr>
<td>6. Rank</td>
<td>0.54</td>
<td>0.74</td>
<td>0.74</td>
<td>—0.70</td>
<td>0.59</td>
<td>—</td>
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</table>

Note. \( n = 25 \); \( r \) values of ±0.25 are not significant. \( r = ±0.50, p < .05 \); \( r \) values of ±0.51 to ±0.60, \( p < .01 \); \( r = ±0.65 \) to ±0.66, \( p < .001 \); \( r > ±0.68, p < .0001 \). Visible trials = number of visible strategy trials/total trials; correct retrieval = number of correct no visible strategy trials/total number of no visible strategy trials; retrieval errors = number of incorrect no visible strategy trials/total number of no visible strategy trials; AS = associative strength, or the probability of retrieving a correct addition answer, from Siegler (1986, p. 8); sum = addend + augend; rank = ranked difficulty for simple addition problems for early elementary school children (Wheeler, 1939).
was termed backup, was coded so as to capture strategy selection and
the accuracy of strategy use, on trials when facts were not correctly
retrieved from long-term memory. If the frequency of counting trials,
both the correct and incorrect verbal counting and counting fingers trials,
was greater than the frequency of incorrect no visible strategy trials then
backup was coded \((\text{number of correct counting trials}) - (\text{number of}
\text{incorrect counting trials})\): otherwise, backup was coded \((0 - (\text{number of}
\text{incorrect no visible strategy trials}))\).

In this way, a high score on the backup variable would indicate the
accurate use of a counting algorithm when no retrieved number met the
accuracy criterion, whereas a low score would indicate frequent guessing.
The backup variable was significantly correlated with the first variable,
\(r = .49, p < .01\), which indicated that subjects who accurately retrieved
facts from long-term memory to solve some problems tended to effectively
employ a counting algorithm to solve more difficult problems. In other
words, a high score on both variables would suggest the subject was
making adaptive strategy choices for solving addition problems, whereas
a low score would suggest the subject’s strategy choices were not related
to problem characteristics (e.g., problem difficulty). Scores on both vari-
ables were transformed to \(Z\) scores and summed to create the composite
strategy choice variable. Two subjects never used the retrieval strategy,
and their scores on the first variable were, therefore, undefined. Thus,
the following analyses excluded the data on these two subjects.

Table 3 presents the zero-order correlations between the strategy choice
variable and standard scores on the Arithmetic subtest of the WRAT
and the 10 WPPSI subtests. Inspection of Table 3 reveals that the strategy
choice variable was significantly correlated with the Arithmetic subtest
of the WRAT and with the Arithmetic, Geometric Design, and Mazes
subtests of the WPPSI. A \(t\) test for dependent \(r\)'s was employed to
determine if the magnitude of the correlation between the strategy choice
variable and each of the ability tests differed significantly (Cohen &
Cohen, 1983).

These \(t\) tests revealed that the magnitude of correlation between the
strategy choice variable and the Arithmetic subtest of the WRAT (.71)
was significantly higher than each of the 10 remaining correlations presented
in Table 3 \((p's < .01)\). The level of correlation between the strategy
choice variable and the Arithmetic (.47), Geometric Design (.52), and
Mazes (.48) subtests of the WPPSI did not differ significantly \((p's > .05)\).
However, the magnitude of the correlation between each of these three
subtests and the strategy choice variable was significantly higher than
the level of correlation between the strategy choice variable and each
of the remaining seven WPPSI subtests \((p's < .01)\).

Inspection of Table 4 reveals significant zero-order correlations between
the strategy choice variable and both Performance IQ and Full Scale IQ.
### TABLE 3

**Correlations between Strategy Choice Variable, Arithmetic Achievement Test, and Wechsler Preschool and Primary Scale of Intelligence Subtests**

<table>
<thead>
<tr>
<th>Strategy Choice Variable</th>
<th>Wide Range Achievement Test Arithmetic</th>
<th>Wechsler Preschool and Primary Scale of Intelligence Subtests</th>
<th>Verbal Scale</th>
<th>Performance Scale</th>
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<tr>
<td></td>
<td>In</td>
<td>Vo</td>
<td>Ar</td>
<td>Sm</td>
</tr>
<tr>
<td>Strategy Choice Variable</td>
<td>.71***</td>
<td>.26</td>
<td>.11</td>
<td>.47*</td>
</tr>
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</table>

*Note.* */* < .01, **/p < .001, ***p < .0001. In, Information; Vo, Vocabulary; Ar, Arithmetic; Sm, Similarities; Cm, Comprehension; AH, Animal House; GD, Geometric Design; BD, Block Design; PC, Picture Completion; Mz, Mazes. Strategy choice variable = Z (number of correct no visible strategy trials/total number of no visible strategy trials) + Z (backup). If the number of counting trials (verbal counting or counting fingers) was greater than the number of incorrect no visible strategy trials then backup was coded (number of correct counting trials) – (number of incorrect counting trials); otherwise, backup was coded 0 – number of incorrect no visible strategy trials.
The significance of these two correlations likely reflects the significant relationship between the strategy choice variable and the Arithmetic, Geometric Design, and Mazes subtests. In all, the results presented in Tables 3 and 4 indicate that individual differences in strategy choices for solving addition equations appear to be related to ability differences in the numerical facility and, perhaps, spatial ability domains.

**Component Scores and Performance on the Ability Measures**

In the final set of analyses, we sought to determine if individual differences in the temporal duration of the memory retrieval process were related to performance on the traditional measures of arithmetic ability, as was found for adults (Geary & Widaman, 1987). First, the protocols for each of the 42 subjects in the study were examined and any subject with less than four correct retrieval trials was excluded from any further analyses \( n = 11 \). Reaction times for the remaining 31 subjects for correct retrieval trials were obtained: total RT’s = 319. Sixteen RT’s were deleted as outliers (greater than 3 SD’s from individual RT means), or due to equipment failure, or because it was noted on the protocol that the subject was distracted during that trial. After the deletion of these RT’s, one subject had less than four correct retrieval trials and was eliminated from any further analyses. The remaining 30 subjects had an average of 10 (SD = 6.5) correct retrieval trials.

Next, average RT for memory retrieval trials was correlated with alternative search/compute structural variables, representing the five counting-based models proposed by Groen and Parkman (1972), the square of the correct sum (Ashcraft & Battaglia, 1978), and the problem’s product (Geary, Widaman, & Little, 1986; Miller, Perlmutter, & Keating, 1984; Widaman, Geary, Cormier, & Little, in press), and with various indices which may be used to represent the associative strength in long-term memory between any given simple addition problem and its correct answer. Specifically, average RT was correlated with the AS values (Siegler, 1986), the rank variable (Hamann & Ashcraft, 1985; Wheeler, 1939), the

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**Table 4**

<table>
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<th>Wechsler Preschool and Primary Scale of Intelligence IQ Scales</th>
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<tr>
<td>Verbal IQ</td>
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<td>-----------</td>
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<tr>
<td>Strategy Choice Variable</td>
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</table>

\(^* p < .01\).
percentage of children who mastered each problem on a learning task (Wheeler, 1939), and a variable reflecting the frequency with which simple addition problems were presented in kindergarten mathematics textbooks (Hamann & Ashcraft, 1986).

The rank variable (Wheeler, 1939) showed the strongest zero-order correlation with average RT, $r = .50, p < .05$. So, we next fit individual regression equations to correct memory retrieval trial RT’s, for each of the 30 subjects, using rank as the independent measure. Intercept values and raw regression weights, which are termed component scores, for the rank variable were obtained from each of the 30 equations.

Finally, hierarchical regression equations were employed to determine if component scores for the rank variable were significantly related to performance on each of the 10 WPPSI subtests, the Arithmetic subtest of the WRAT, and the three IQ scales. For each of the resulting 14 equations, the strategy choice variable was entered first, followed by the rank variable, and the intercept values. Partial $F$ ratios for the rank and intercept measures were examined to determine if scores for these two variables were uniquely related to performance on the ability measures. In these equations, the partial $F$ ratio tested the significance of the variance explained by the variable, independent of all variables entered beforehand in the equation (Cohen & Cohen, 1983). These analyses indicated that the intercept values were not significantly related to performance on any of the ability measures ($p$'s > .10). So, the intercept variable was dropped and the equations were recomputed.

This final set of analyses indicated that component scores for the rank variable were significantly related to performance on the Arithmetic subtest of the WPPSI, $F(1, 27) = 6.38, p < .05; R = .57, \beta = -.41$. Moreover, a marginally significant relationship between component scores on the rank variable and performance on the Arithmetic subtest of the WRAT was found, $F(1, 27) = 3.87, p = .059; R = .83, \beta = -.22$. Scores for the rank variable were not significantly related to performance on any of the remaining nine ability tests or to the three IQ scales ($p$'s > .05).

**GENERAL DISCUSSION**

The primary purpose of the present study was to replicate and concurrently assess the external validity of the strategy choice model proposed by Siegler and his colleagues (Siegler, 1986; Siegler & Shrager, 1984; Siegler & Taraban, 1986). In general, the substantive predictions generated by the model were supported empirically by the results of this study. The strategies employed by the preelementary school children to solve simple addition equations were the same as those identified by Siegler and Robinson (1982), although the fingers strategy occurred rather infrequently, as was found by Baroody (1987).

The proposed hierarchical structure of strategy selection was generally
supported by the temporal characteristics of each of the four strategies, as well as by the finding that the probability of correct retrieval was strongly and inversely related to the frequency of overt strategy use. For these data, adaptive strategy selection would be best characterized as memory retrieval followed by the use of a backup counting strategy (either counting fingers or verbal counting) when memory retrieval failed to produce a satisfactory answer, although the fingers strategy would appear to be an intermediate strategy for some children. Finally, the results of this study, combined with Siegler’s (1988a) recent findings, suggest that some children do not make adaptive strategy choices. Rather, these children appear to rely on the retrieval strategy even when the retrieved answer is likely to be incorrect.

With regard to external validity, the significant correlations between the variable representing strategy choices in addition and performance on both of the measures of arithmetic ability provided support for the convergent validity of the processes identified in the strategy choice model. Individual differences in strategy choices, as defined by Siegler’s (1986) model, for solving simple addition problems are related to performance on traditional measures of arithmetic ability. The finding that scores on the strategy choice variable were more strongly related to performance on the Arithmetic subtest of the WRAT than to performance on the Arithmetic subtest of the WPPSI might have been due to the relative difficulty of these two measures.

Items on the Arithmetic subtest of the WRAT, for preschool children, assess skills more basic or equivalent to those skills needed to perform successfully on the experimental task (i.e., for the solution of the 25 addition problems), whereas many of the items comprising the Arithmetic subtest of the WPPSI require skills more advanced, e.g., the mental solution of arithmetic word problems, than those number skills required by the experimental task. Thus, the significant difference in the level of correlation between the strategy choice variable and the two arithmetic tests was likely due to the fact that items on the Arithmetic subtest of the WRAT were more similar to the items on the experimental task than were items on the Arithmetic subtest of the WPPSI.

Regardless, the finding of a significant relationship between strategy choices for solving simple addition problems, the speed of retrieving addition facts from long-term memory, and performance on the rather more complex problems comprising the Arithmetic subtest of the WPPSI is noteworthy and consistent with Kaye’s (1986) argument that the mastery of basic numerical operations and facts facilitates the acquisition of more complex mathematical concepts and procedures.

Support for the discriminant validity of Siegler’s (1986) model for the representation of strategy choices in addition was modest. Individual differences in strategy choices for solving addition problems were not
related to performance on measures of language-related ability. However, the strategy choice variable was significantly correlated with two measures of spatial-related ability, the Geometric Design and Mazes subtests of the WPPSI. The former subtest requires the reproduction of both simple and complex geometric designs, whereas the latter, Mazes, subtest requires spatial scanning.

This finding of a relationship between strategy choices in addition and performance on two tests of spatial-related ability is consistent with previous studies, which have often shown that mathematical and spatial ability covary (e.g., Pattison & Grieve, 1984). Although it is unlikely that the same component processes underlie performance on mathematical and spatial tasks (Linn & Petersen, 1985), this result would suggest that the relationship between mathematical and spatial ability exists before formal academic training.

Moreover, the finding that strategy choices in solving addition equations were related to performance on two spatial measures but that the temporal duration of fact retrieval was not related to spatial ability suggests that the early reliance on spatial-related information and strategies, e.g., combining arrays of objects to add (Carpenter & Moser, 1984), to solve arithmetic problems might contribute to the observed relationship between performance on numerical and spatial ability measures (Luria, 1980).

The componential analyses indicated that the speed of information retrieval from a network of stored arithmetic facts was inversely related to performance on the traditional tests of arithmetic ability, as was found for adults (Geary & Widaman, 1987). The more readily information was retrieved from memory the better the performance on the ability measures. In all, the componential analyses coupled with the relationship between strategy choices and the ability measures and coupled with earlier findings (Geary & Widaman, 1987; Geary et al., 1987; Goldman, Pellegrino, & Mertz, 1987; LeFevre & Bisanz, 1986) suggest a convergence in the processes identified in information-processing models and operations assessed by traditional measures of basic mathematical ability.

Finally, the results of the present study suggest that the relationship between variables representing component processes, or strategy choices, and performance on reference ability measures may change with skill development. The source of individual differences on the Arithmetic subtest of the WRAT, for preelementary school children, was more strongly related to individual differences in strategy selection than to individual differences in the speed of executing a component process. Performance on the more difficult Arithmetic subtest of the WPPSI was related to both strategy choices and to the speed of fact retrieval.

On the other hand, adults rely primarily on fact retrieval to solve addition problems (Svenson, 1985), and for adults individual differences in the speed of executing two elementary component processes, memory
retrieval and the carry operation, for solving arithmetic problems would appear to underlie individual differences on traditional numerical facility measures (Geary & Widaman, 1987). Thus, early in the acquisition of numerical skills, adaptive and task appropriate strategy choices might be a primary source of individual differences in mathematical ability, but as strategy choices become unnecessary, due to an increased reliance on the memory retrieval of facts and procedures, the speed of executing component processes would appear to underlie ability differences on numerical tests.

To conclude, the present study replicated and provided support for the external validity, both convergent and discriminant forms, of Siegler's (1986) model for representing strategy choices for solving addition problems. Future research will need to assess the external validity of the model for cognitive domains other than addition. Finally, the present results suggest that the relationship between variables representing information-processing components, or strategies, and performance on traditional ability measures may vary with individual skill and across development.

REFERENCES


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